LINEAR FEATURE DETECTION IN SAR IMAGES

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ABSTRACT

The problem treated in this paper is the one of detection and restoration of ship wakes in Synthetic Aperture Radar (SAR) images. This cannot be easily made, because SAR images are corrupted by a granular noise, called speckle and because there is no information about the direction and the level of the wake. For these reasons, most detection algorithms use the Radon transform, which makes square a straight with a point. We propose here a new method, based on the mariage between the Radon transform and a filtering method used to interpolate the image in a rotating reference system, introduced by the Radon transform theory. This filtering technic is the stochastic matched filtering technic, which allows to maximize the signal to noise ratio after processing. Experimental results on SIR-C/X-SAR images are presented and compared to those obtained using the classical approach.

1. INTRODUCTION

Recently, a great deal of research has been dedicated to ship wake detection in Synthetic Aperture Radar (SAR) images. Indeed, it is well known that SAR images are able to show ship wakes as lines darker or sometimes brighter than the surrounding sea. Most of the detection algorithms use the Radon transform [2, 7, 8]. Indeed, when an image contains a straight line or a segment, its Radon transform exhibits a narrow peak if the line is brighter than its surroundings and a trough in the opposite case. Thus, the problem in finding lines is related to detect these peaks and troughs in the transform domain. Other methods use the detection on both ships and ship wakes [5]. Given that SAR images are affected by a granular, multiplicative noise (called speckle), most of these detection algorithms pre-filter the data in order to improve the visibility of the ship wakes.

We know that the application of the Radon transform requires the computation of interpolated image in a rotating reference system. In this paper, we propose a new method based on the mariage between the Radon transform and a filtering method. We use this filtering technic to compute the interpolations of the SAR image, in order to estimate properly the signal of interest (the ship wake) in the rotating reference system. This processing is called the stochastic matched filtering technic [1]. It is based on the signal expansion into series of functions with uncorrelated random variables for decomposition coefficients. This corresponds to the Karhunen-Loeve expansion in the case of a white noise. Because the chosen basis functions improve the signal to noise ratio after processing, there is no more sinusoidal curves corresponding to the speckle in the Radon domain, and the detection of the peak (or trough) corresponding to the ship wake is improved.

First of all, we recall in section 2 the discrete Radon transform [4], in the case of a two-dimensional signal. Then, we present, in section 3, an interpolation-filtering method for noise corrupted image in a rotating reference system. First, we recall the stochastic matched filtering technic and then we describe how to perform an interpolation based on this method and using the discrete cosine transform. We finish this section with the explanation of the subimage processing. Next, in section 4, we propose an example of application of our processing on a SIR-C/X-SAR image, which shows a moving ship and its dark turbulent wake. We finish this article with a comparison of our results with those obtained by the classical approach, which uses the Radon transform based on the nearest neighbor interpolation.

2. THE DISCRETE RADON TRANSFORM

The Radon transform on Euclidean space was first established by Johann Radon in 1917. Nearly half a century after Radon's work, the Hough transform for detecting straight lines in digital pictures was introduced. But this transform is actually a special case of the Radon transform. We are going to recall in this section the discrete Radon transform of a two-dimensional signal.

Considering an image I, $(M + 1) \times (M + 1)$ pixels, its discrete Radon transform, $\widehat{I}$ is expressed by:

$$\widehat{I}(x_\theta, \theta) = \sum_{y_\theta = -M/2}^{M/2} I(x_\theta \cos\theta - y_\theta \sin\theta, x_\theta \sin\theta + y_\theta \cos\theta),$$

where x_θ and y_θ are integers which are bounded by $-\frac{M}{2}$ and $\frac{M}{2}$; θ corresponds to the rotation angle and takes values between 0 and π.

The previous equation shows that the computation of the Radon transform requires, for each parameter θ, the calculation of the new pixel values in reference system $\Re_\theta$. Indeed, the new coordinates are not integer values and so are not corresponding to the native mesh of the image.

By nature, the Radon transform accentuates linear features in an image by integrating along all possible lines. The result is that an image which is non zero in a single point (x_0, y_0) has a Radon transform which is non zero along a sinusoidal curve of equation $x_\theta = x_0 \cos \theta + y_0 \sin \theta$. The phase and frequency of this sinusoidal curve depend on the spatial location of the corresponding point in the original image. If the original image contains a straight line or a segment, its Radon transform exhibits a narrow peak, if the line is brighter than its surrounding, and a trough in the opposite case. The coordinates of the peak (or trough) are (x_{θ_0}, θ_0) which correspond to the parameters of the polar equation $x_{\theta_0} = x \cos \theta_0 + y \sin \theta_0$ of the straight line. Thus, the problem of finding lines is reduced to the detection of peaks and troughs in the transform domain. The Radon transform is particularly suited for finding lines in a noise-corrupted image, because the integration process tends to cancel out intensity fluctuations due to the noise. For this reason, we can find in the literature several applications in the domain of wakes detection (see [2, 7, 8] for example).

3. INTERPOLATION-FILTERING OF A ROTATING IMAGE

We have seen that the Radon transform of an image implies, for each value of the θ parameter, the computation of an interpolated image. We want to use the Radon transform in order to detect ship wakes in SAR images. Given that SAR images are corrupted by a granular noise, called speckle, it is of great interest to take into account this noise to compute the interpolation of such an image, in order to give a good estimation of the signal of interest in the rotating reference system. For this reason, we present here an interpolation method, based on the stochastic matched filtering method, which principle is to expand the noise-corrupted signal into series of functions with uncorrelated random variables for decomposition coefficients.

3.1. The stochastic matched filtering method

Consider a two-dimensional noise-corrupted signal, $Z(x, y)$, defined over $D = [-T, T] \times [-T; T]$. This one corresponds to the superposition of a signal of interest $S(x, y)$ with a noise $B(x, y)$:

$$Z(x, y) = S(x, y) + B(x, y) \qquad \forall (x, y) \in D,$$

where $S(x, y)$ and $B(x, y)$ are assumed to be independent and stationary.

We want to expand simultaneously the signal of interest and the noise into series of the form:

$$\begin{cases} S(x, y) = \lim_{N \to \infty} \sum_{n=1}^{N} s_n \Psi_n(x, y) \\ B(x, y) = \lim_{N \to \infty} \sum_{n=1}^{N} b_n \Psi_n(x, y), \end{cases}$$

In these expressions, $\Psi_n(x, y)$ are the deterministic linearly independent basis functions, and s_n and b_n represent zero-mean, random variables expressed by the following relations:

$$\begin{cases} s_n = \iint_D S(x, y) \Phi_n(x, y) dx dy \\ b_n = \iint_D B(x, y) \Phi_n(x, y) dx dy. \end{cases} \tag{1}$$

The determination of these random variables depends on the choice of the set of deterministic functions $\{\Phi_n(x, y)\}$. We will used the set, which provides the uncorrelation of the random variables, i.e.:

$$\begin{cases} E\{s_n s_m\} = E\{s_n^2\} \delta_{n,m} \\ E\{b_n b_m\} = E\{b_n^2\} \delta_{n,m}. \end{cases}$$

Now, we show how to determine these functions $\Phi_n(x, y)$. In order to find them, let us consider the stochastic matched filtering technic [1].

If we consider a deterministic, stationary two-dimensional signal, called $S(x, y)$, which is defined over D, corrupted by an ergodic, stationary noise $B(x, y)$, the matched filtering technic consists in finding a function $\Phi(x, y)$, defined over D, in order to maximize the signal to noise ratio K, expressed by the following relation:

$$K = \frac{\left| \iint_D S(x, y) \Phi(x, y) dx dy \right|^2}{E\left\{ \left| \iint_D B(x, y) \Phi(x, y) dx dy \right|^2 \right\}}.$$

When the signal is not deterministic, but a random, zero-mean, stationary, two-dimensional signal $S(x, y)$, we can show that K can be explained as follows:

$$K = \frac{E\left\{ \left| \iint_D S(x, y) \Phi(x, y) dx dy \right|^2 \right\}}{E\left\{ \left| \iint_D B(x, y) \Phi(x, y) dx dy \right|^2 \right\}}.$$

Given that this signal to noise ratio can be rewritten as the ratio of two quadratic forms, it appears to be a Rayleigh quotient, so it will be maximized if $\Phi(x, y)$ is the two-dimensional eigenfunction associated to the maximal eigenvalue of the following integral equation:

$$\iint_D \Gamma_{SS}(x - x', y - y') \Phi_n(x', y') dx' dy' = \\ \lambda_n \iint_D \Gamma_{BB}(x - x', y - y') \Phi_n(x', y') dx' dy', \tag{2}$$

for all $(x, y) \in D$ and where Γ_{SS} and Γ_{BB} represent the covariances of the signal and of the noise, respectively.

Random variables s_n and b_n are uncorrelated, when the $\Phi_n(x, y)$ functions are the eigenfunctions of integral equation (2), with eigenvalues λ_n verifying:

$$\lambda_n = \frac{E\{s_n^2\}}{E\{b_n^2\}}.$$

When eigenfunctions $\Phi_n(x, y)$ are normalized such as the following integral

$$\iint_D \iint_D \Gamma_{BB}(x - x', y - y') \Phi_n(x, y) \Phi_n(x', y') dx dx' dy dy'$$

takes one for value, we can show that functions $\Psi_n(x, y)$ are expressed by:

$$\Psi_n(x, y) = \iint_D \Gamma_{BB}(x - x', y - y') \Phi_n(x', y') dx' dy'. \tag{3}$$

In these conditions and considering the z_n random variables, obtained by projecting functions $\Phi_n(x, y)$ on noise-corrupted signal $Z(x, y)$, we can show that the use of the following expansion

$$Z(x, y) = \lim_{N \to \infty} \sum_{n=1}^{N} z_n \Psi_n(x, y)$$

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corresponds to a signal to noise ratio of the nth component of $Z(x, y)$ expressed by:

$$\frac{\sigma_S^2}{\sigma_B^2}\lambda_n,$$

where σ_S^2/σ_B^2 is the signal to noise ratio before processing.

So all the eigenfunctions $\Phi_n(x, y)$ associated to eigenvalues λ_n greater than one can contribute to an improvement of the signal to noise ratio. For this reason, filtering the observed signal can be made by keeping all the components with a signal to noise ratio greater to a certain level, anyhow greater than 1.

3.2. Interpolation using the stochastic matched filtering method

To compute an interpolation based on the stochastic matched filtering method, the basic idea is to expand the observed signal and then to restore the signal of interest using the $\Psi_n(x, y)$ functions previously interpolated. But this reasoning presents some defaults. Indeed, it implies an heavy CPU budget and some memory problems may appear. For these reasons, we are going in this section to propose a new formulation for the stochastic matched filtering method by using the discrete cosine transform.

3.2.1. Analytical approximation for the solutions of the integral equation

We can find in the literature several works based on the stochastic matched filtering technic in its discrete form (see [6] for example). Unfortunately, when we consider the discrete form of integral equation (2), the eigenvectors solution of this generalized eigenvalue problem are linked to the native increment of the image and could cause problems for image interpolation. So, we do not consider the discrete relation but the continuous one to find the $\Phi_n(x, y)$ functions.

Considering the DCT[1] coefficients $\alpha_{k,l}^n$, $\Omega_{k,l,p,q}^{\Gamma_{SS}}$ and $\Omega_{k,l,p,q}^{\Gamma_{BB}}$ of functions $\Phi_n(x, y)$, $\Gamma_{SS}(x - x', y - y')$ and $\Gamma_{BB}(x - x', y - y')$, we can show that solving integral equation (2) becomes equivalent to solving the following linear system:

$$\sum_{k=0}^{Nf}\sum_{l=0}^{Nf}\alpha_{k,l}^n\Omega_{k,l,p,q}^{\Gamma_{SS}} = \lambda_n\sum_{k=0}^{Nf}\sum_{l=0}^{Nf}\alpha_{k,l}^n\Omega_{k,l,p,q}^{\Gamma_{BB}}, \qquad (4)$$

where $(Nf + 1)$ corresponds to the number of DCT coefficients taken account and is high enough to ensure the uniform convergence of the series to their respective functions.

Finally, the analytical approximation $\widehat{\Phi}_n(x, y)$ of the $\Phi_n(x, y)$ functions solution of the integral equation, is obtained with the following relation, for $(x, y) \in D$:

$$\widehat{\Phi}_n(x, y) = \sum_{k=0}^{Nf}\sum_{l=0}^{Nf}\alpha_{k,l}^n \cos\left(\frac{\pi k(x - T)}{2T}\right)\cos\left(\frac{\pi l(y - T)}{2T}\right).$$

This new method for finding an analytical approximation for the solutions of the integral equation has been quantified and compared to the classical approach, in the case of the Fredholm integral equation, that is when the noise covariance describes a white noise [3].

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[1]Discrete Cosine Transform

3.2.2. Interpolation-filtering method

We propose in this subsection a new formulation of the stochastic matched filtering method by using the discrete cosine transform. In these conditions, we are looking for the expression of the coefficients of the filtered signal expanded into cosine series; we shall reconstruct the approximation of the restored signal in the final phase of the processing.

Let $Z(x, y)$ be the observed signal to be expanded and let $\widehat{Z}(x, y)$ be the reconstructed filtered signal. We have:

$$\widehat{Z}(x, y) = \sum_{n=1}^{Q} z_n\Psi_n(x, y) \qquad \forall(x, y) \in D, \qquad (5)$$

where Q is chosen such as λ_Q is greater to a certain threshold, anyhow greater than 1.

In the last relation, z_n are the random variables to be determined from the input data:

$$z_n = \iint_D Z(x, y)\Phi_n(x, y)dxdy. \qquad (6)$$

We have seen that these random variables are uncorrelated when functions $\Phi_n(x, y)$ are the eigenfunctions of integral equation (2). The $\Psi_n(x, y)$ basis functions are obtained by projecting functions $\Phi_n(x, y)$ on the noise covariance as described in relation (3). First, we modify this relation, in order to express the $\beta_{p,q}^n$ coefficients of the $\Psi_n(x, y)$ cosine series. It comes:

$$\beta_{p,q}^n = T^2\sum_{k=0}^{Nf}\sum_{l=0}^{Nf}\alpha_{k,l}^n\Omega_{k,l,p,q}^{\Gamma_{BB}}, \qquad (7)$$

for $(p, q) = 0, 1, \ldots, Nf$.

In like manner, from expansion (5), we obtain for expression of the $\widehat{\vartheta}_{k,l}$ DCT coefficients of restored signal $\widehat{Z}(x, y)$:

$$\widehat{\vartheta}_{k,l} = \sum_{n=1}^{Q} z_n\beta_{k,l}^n. \qquad (8)$$

To end, we have to explain the z_n coefficients in terms of the coefficients of the observed signal and of the eigenfunctions. We can show that relation (6) is equal to:

$$z_n = T^2\sum_{p=0}^{Nf}\sum_{q=0}^{Nf}\vartheta_{p,q}\alpha_{p,q}^n, \qquad (9)$$

where $\vartheta_{p,q}$ represent the $Z(x, y)$ DCT coefficients.

Considering the θ rotation angle, it is possible to compute the interpolated-filtered signal using the following relation:

$$Z_\theta(x_\theta, y_\theta) = \sum_{k=0}^{Nf}\sum_{l=0}^{Nf}\widehat{\vartheta}_{k,l}\cos\left(\frac{\pi k(x_\theta - T)}{2T}\right)\cos\left(\frac{\pi l(y_\theta - T)}{2T}\right),$$

where (x_θ, y_θ) represents the new coordinates of the pixels in rotating reference system $\Re_\theta$.

With such a formulation, it is possible to use the stochastic matched filtering method for image interpolation in a very short time, because all the computations can be made by using only the algorithm of fast discrete cosine transform.

3.3. Subimages processing

To apply the stochastic matched filtering method, it is necessary to respect the stationary condition for the signal and the noise. We know that we cannot consider an image as a realization of a stationary process. But after segmentation, we can define several areas representative of a texture. So a particular area of the image (a subimage) can be considered as a stationary process. For this reason, we are going to cover the noise-corrupted image with subimages. The subimage size is chosen such as the subimages are assumed to be a texture. For each angle θ, we apply the proposed processing on subimages, with $M \times M$ pixels. Scanning all the image allows a complete processing.
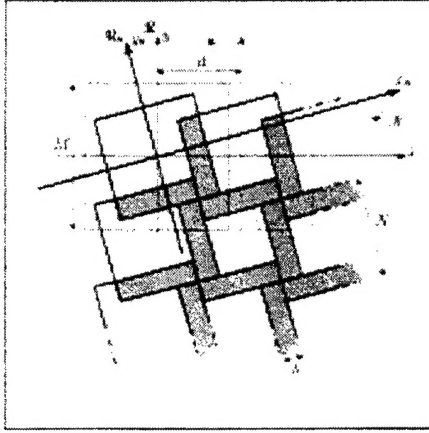
Figure 1: Subimages processing showing the overlapping between adjacent subimages

When we compute the interpolation of a subimage, we find an edge effect. Indeed, in rotating reference system $\Re_\theta$, the coordinates of the pixels localized on the edge of the subimage do not depend on the native subimage in reference system $\Re$. This edge effect being maximal for angle θ equal to $\frac{\pi}{4}$, the subimage size after processing is $N \times N$, with N equal to $\frac{M}{\sqrt{2}}$. To limit these edge effects, it is necessary to segment the native image to obtain subimages which overlap, as shown in figure 1. The gray areas correspond to the superposition of subimages. d and h coefficients represent the distance between the center of two adjacent subimages (in line or row) and the width of the overlapping area respectively. We have:

$$d = \frac{N}{\cos\theta + \sin\theta} \text{ and } h = \left| \frac{N\sin\theta}{\sin\theta + \cos\theta} \right| .$$

We now apply the interpolation-filtering method, proposed in the previous section, to each zero-mean subimage. Assuming that the noise is high-frequency compared to the signal, when we apply this processing to the whole noise-corrupted image with the same number Q of basis functions for each subimage, the resulting image may be smoothed or still noise-corrupted. Indeed, the signal to noise ratio is not the same for each area of the image. For this reason, we are going to process each subimage, with different number Q of basis functions. To find this number, let us consider mean square error $\bar{\epsilon}$ between reconstructed signal $\widehat{Z}(x,y)$ and signal of

interest $S(x,y)$. We can show:

$$\bar{\epsilon} = \sigma_S^2 + \frac{1}{4}\sum_{n=1}^{Q} \left(\sigma_B^2 - \lambda_n\sigma_S^2\right)\sum_{k=0}^{Nf}\sum_{l=0}^{Nf}\left(\beta_{k,l}^n\right)^2 .$$

So, for each subimage, we compute $\bar{\epsilon}$ for different values of parameter Q (Q being in the interval $[1; Q_{max}]$, such as $\lambda_{Q_{max}}$ greater than 1). We only keep the Q values, which minimize the mean square error. The noise power, σ_B^2, will be computed in a homogeneous area of the whole noise-corrupted image and the signal power, σ_S^2, will be estimated in the subimage to be processed. Considering now the problem posed by the overlapping areas, the corresponding processed pixels will be computed by averaging each pixel having the same position in the overlapping subimage.

4. SHIP WAKES DETECTION

To illustrate our processing, we have chosen to apply it to an image acquired by the Spaceborne Imaging Radar-C/X-band Synthetic Aperture Radar (SIR-C/X-SAR), which shows a moving ship and its dark turbulent wake. This image is presented figure 2.
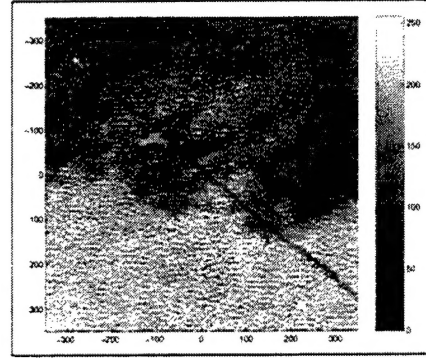

Figure 2: SIR-C/X-SAR image (698 × 698 pixels)

The image size is 698 × 698 pixels. The number of gray levels is 256 (0: black, 255: white). The dark patches in the upper right of this image correspond to smooth areas of low wind. The ship's wake is about 28 kilometers (17 miles) long in this image and investigators believe that may reveal that the ship is discharging oil. Classically, to quantify the perturbation level of a SAR image, we determine its speckle level. This one is obtained by computing the variation coefficient (C in the following), obtained on several homogeneous areas of the image. Let W be the number of homogeneous areas I_n, we have:

$$C = \frac{1}{W}\sum_{n=1}^{W}\frac{\sigma_n}{E\{I_n\}} .$$

For the studied image, the variation coefficient is equal to 0.277. We have now enough information about the studied image to process it.

4.1. Signal and noise auto-correlation functions

We have seen that the interpolation-filtering method, presented in section 3, requires the *a priori* knowledge of the signal and the

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noise auto-correlation functions. In order not to favor any particular wake orientation, we have chosen to represent the signal auto-correlation function by an isotropic model. This one results, for different θ values, from the averaging of several auto-correlation function computations of a two colors straight line placed in a rotating reference system. This model is presented figure 3.a.
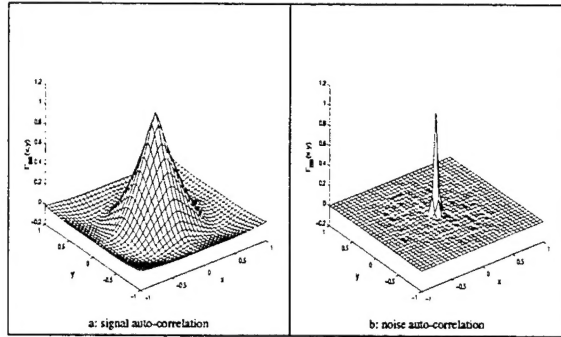

a: signal auto-correlation   b: noise auto-correlation

Figure 3: Normalized model for the signal and noise auto-correlation functions ($T = 1$)

The noise auto-correlation function is presented in figure 3.b. This model has been obtained by averaging several realizations of noise auto-correlation functions computed in some homogeneous areas of the native image.

4.2. Radon domain and wakes restoration

After zero-meaning the image presented figure 2, we have processed it with a subimage size equal to 17×17 pixels, to respect the coherence length of the noise.

We present in figure 4 the interpolated-filtered image for angle θ equal to $35°$. For this image, number Q of basis functions is included between 1 and 13 depending on the native signal to noise ratio of the subimage to be processed (near 13 basis functions for the restauration of the wake and 1 basis function for the rest of the image).
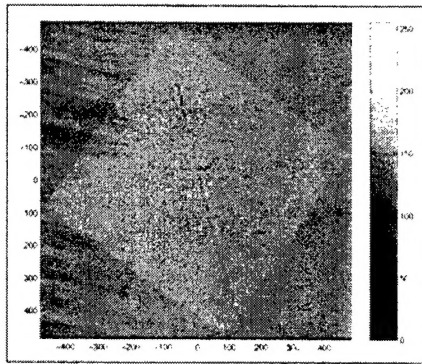

Figure 4: SIR-C/X-SAR image in reference system $\Re_{35}$, (973 × 973 pixels)

Analyzing this figure, we see that the proposed processing allows

a great reduction of the speckle and a good restitution of the signal of interest (the wake). Indeed, there is no more dark patches and the variation coefficient is now equal to 0.016, so there is an improvement by a factor of 18 of the speckle level.

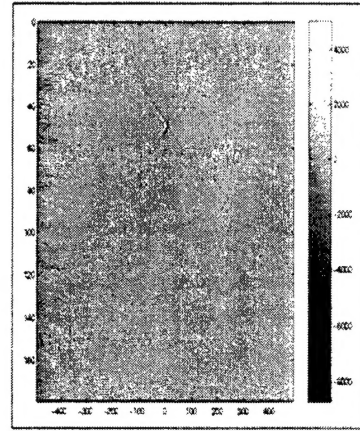We present in figure 5 the resulted image in the Radon domain.

Figure 5: Radon domain obtained using the interpolation-filtering method

In the transform domain, the vertical axis represents the orientation of each integration line, while the horizontal axis represents the distance of each line from the center of the image. The trough corresponding to the wake is clearly evident. Its vertical position is near $51°$ and corresponds to the orientation of the wake. Furthermore, several sinusoidal curves of poor amplitude regarding to the trough amplitude are visible in the Radon domain and correspond to the Radon transform of the residual perturbations after interpolation-filtering.

From this transform domain, we have used the inverse Radon transform to find the location of the wake in the spatial domain. This inverse transform has been applied to the image, presented in figure 5, before raised to the power of three in order to improve the amplitude of the trough in regards to the rest of the image. The different interpolations have been made using a nearest neighbor interpolation. We present in figure 6 the resulted image.
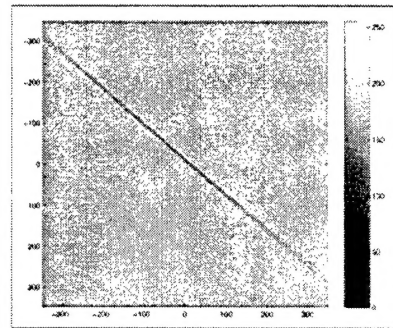

Figure 6: Restored wake from figure 5 (696 × 696 pixels)

The resulted image shows that our processing allows a great im-

provement of the wake readability. All the disruptive pixels have disappeared. The variation coefficient for this image is equal to 0.002 compared to 0.277 for the original image.

5. COMPARISON WITH CLASSICAL PROCESSING

We present figure 7 the transform domain of the SIR-C/X-SAR image. Each image in the rotating reference system has been computed using a nearest neighbor interpolation.
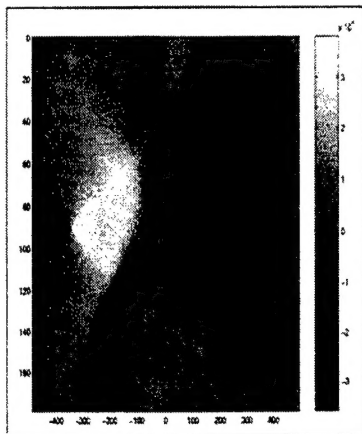

Figure 7: Radon domain obtained using the classical approach

In the transform domain, we can see several overlapping sinusoidal curves. They are due to the presence of the speckle in the native image and they limit the visibility of the trough corresponding to the wake. Furthermore, the dark patches in the original image create in the transform domain a large sinusoidal curve with an amplitude higher than the one of the trough. So, using a threshold to extract the trough will be misleading, because the minima of the transform domain do not correspond to the trough. In this case, the detection cannot be released with this processing.
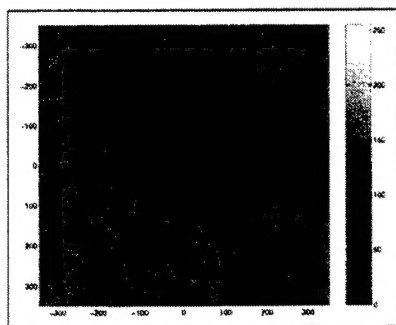

Figure 8: Restored wake from figure 7 (698 × 698 pixels)

To illustrate the previous remarks, we present in figure 8 the result obtained after having applied the inverse Radon transform, with the same process as for the image presented in figure 6. It confirms that this processing is not efficient for such an image. Indeed, we remark a lot of perturbations, mainly due to the presence of the dark patches. The variation coefficient for this image is equal to 0.145 and the improvement is only a factor of 2.

6. CONCLUSIONS

We have presented in this paper a new processing which allows ship wakes detection in SAR images. This processing is based on the computation of the SAR image Radon transform. The original contribution of this work, compared to the classical approaches in this domain consists in taking into account the noise for the image interpolation in the rotating reference system. This allows the perturbations to have a lower impact in the transform domain, the corresponding sinusoidal curves having an amplitude smaller than the peak or trough characteristic of the wake. We have compared the transform domain and the restored wake obtained by our processing on SAR images, with those obtained with the classical processing. In all cases, our processing presents far better results. With our processing, the probability of false alarm or no detection is lower than with the classical approach, because only the signal of interest is considered.

An important drawback of the Radon transform is that it is global by nature, so this transform cannot tell the difference between long and short straight lines. For this reason, future work in this domain concerns the application of the proposed interpolation method for the computation of the localized Radon transform [2], which allows to localize the beginning of the wake.

7. REFERENCES

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